Journal of Engineering Physics and Thermophysics, Vol. 79, No. 3, 2006

DETERMINATION OF FILTRATION PARAMETERS OF A LAYERED BED USING DATA OF UNSTEADY LIQUID INFLOW TO A WELL

E. R. Badertdinova[a] **and M. Kh. Khairullin**[b]

UDC 517.958:532

A computational algorithm based on the theory of ill-posed problems is proposed for evaluating filtration parameters of a layered bed using the results of nonstationary hydrodynamic investigations of vertical wells.

Methods for determining the filtration parameters of a bed from the results of nonstationary hydrodynamic investigations of wells are widely employed in practice. They are based on studying unsteady processes of the pressure redistribution after a well is started or stopped. In the current work, consideration is given to the problem of determining filtration parameters of the layered bed from the results of nonstationary hydrodynamic investigations of wells using regularization methods. After a producing well is closed, an unsteady liquid inflow from the seams to the opening persists for some time so that pressure is recovered. The duration and character of the inflow depend on the pressure redistribution and filtration characteristics of each seam. Therefore, the recording of the curve of the decrease in the discharge of each seam as a result of the pressure recovery provides information needed for estimating filtration properties of the seam [1, 2].

The problem of determining the hydraulic conductivity for the layered bed is formulated as follows: find $\sigma = (\sigma_1, ..., \sigma_{2n-1})$ when the filtration process is described by the system of equations in the multiply connected region D with boundary $\partial D = G + \Gamma$

$$\beta^* H_1 \frac{\partial p_1}{\partial t} + L_1 p_1 + \omega_1 (p_1 - p_2) = 0 ,$$

$$\beta^* H_3 \frac{\partial p_2}{\partial t} + L_2 p_2 + \omega_1 (p_2 - p_1) + \omega_2 (p_2 - p_3) = 0 ,$$

$$\dots \tag{1}$$

$$\beta^* H_{2n-1} \frac{\partial p_n}{\partial t} + L_n p_n + \omega_{n-1} (p_n - p_{n-1}) = 0 , \quad 0 < t \le T ,$$

with the initial and boundary conditions

$$\int_\Gamma \sigma_{2k-1} \frac{\partial p_k}{\partial n} ds = q_k (t) , \quad \left. \frac{\partial p_k}{\partial \tau} \right|_\Gamma = 0 , \quad p_k \big|_G = p_{k0} , \quad p_k (x, y, 0) = \varphi_k , \quad k = 1, 2, ..., n , \tag{2}$$

where $L_k p_k = -\text{div} (\sigma_{2k-1} \text{ grad } p_k)$, $k = 1, 2, ..., n$; $\omega_k = \sigma_{2k}/H_{2k}^2$, $k = 1, 2, ..., n-1$; $c_1 \le \sigma_k \le c_2$, $k = 1, 2, ..., 2n-1$.

Additionally, bottom-hole pressures $p_k^{(b)}(t)$, measured on the well, are assumed to be known. This inverse problem gives rise to an implicitly specified nonlinear operator

[a]Kazan State Technological University, 68 K. Marx Str., Kazan, 420015, Russia; [b]Kazan Institute of Mechanics and Mechanical Engineering, Kazan Scientific Center of the Russian Academy of Sciences, 2/31 Lobachevskii Str., Kazan, 420111, Russia; email: khairullin@mail.knc.ru. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 79, No. 3, pp. 128–130, May–June, 2006. Original article submitted June 17, 2004; revision submitted April 27, 2005.

1062-0125/06/7903-0553

$$A\sigma = P^* , \tag{3}$$

where

$$P^* = \left\{p_k^*\right\}, \quad p_k^* = \int_0^T p_k^{(\text{ç})}(t)\, dt , \quad k = 1, 2, ..., n .$$

Generally, the quantity P^* is not accurately known, i.e., $\|P^* - P_\delta^*\| \le \delta$, where $\|...\|$ is the norm in the Euclidean space R^n. Problem (3) in the variational formulation reduces to the minimization of the smoothing functional

$$M^\alpha(\sigma) = \|A\sigma - P_\delta^*\|^2 + \alpha\Omega(\sigma) ,$$

where $\Omega(\sigma) = \sum_{i=1}^{2n-1} (\sigma_k - \sigma_k^0)^2$, $\alpha = \alpha(\delta)$ is the regularization parameter that is in agreement with the measurement error.

Successive approximations of σ^m are constructed as follows: in the neighborhood of σ^m at a fixed value of the regularization parameter $\alpha = \alpha_m$, the nonlinear operator $A\sigma$ is represented as

$$A\sigma = A\sigma^m + A_\sigma'(\sigma^m)(\sigma - \sigma^m) + o(\|\sigma - \sigma^m\|) , \tag{4}$$

where $A_\sigma'(\sigma^m)(\sigma - \sigma^m)$ is the Frechet differential, which is calculated using methods of perturbation theory. Functional (4) is minimized using the Gauss–Newton procedure

$$M^{\alpha_m}(\sigma) = \|A\sigma^m - A_\sigma'(\sigma^m)(\sigma - \sigma^m) - P_\delta^*\|^2 + \alpha_m\Omega(\sigma) .$$

An explicit expression of the Frechet functional can be obtained as in [3]:

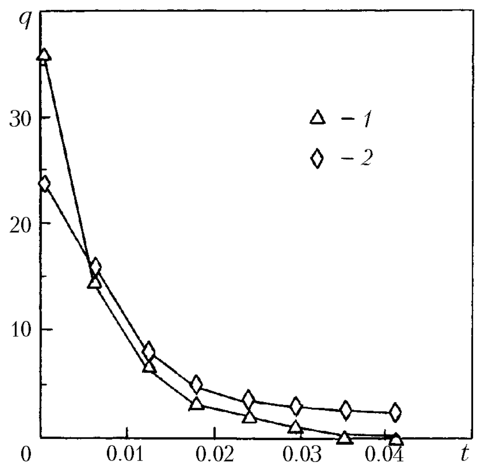
$$A_\sigma'(\tilde{\sigma})(\sigma - \tilde{\sigma}) = (A_i)_{i=1}^n ,$$

$$A_i = \sum_{k=1}^{n} \int_0^T \left(\delta\sigma_{2k-1}\, \text{grad}\, p_k , \text{grad}\, \tilde{p}_k^i\right) dt + \sum_{k=1}^{n-1} \delta\omega_k \int_0^T \left[\left(p_k - p_{k+1}, \tilde{p}_k^i\right) - \left(p_{k+1} - p_k, \tilde{p}_{k+1}^i\right)\right] dt .$$

Here $\tilde{p}^i = (\tilde{p}_1^i, \tilde{p}_2^i, ..., \tilde{p}_n^i)$ is the solution of the corresponding related problem at the hydraulic coefficient $\tilde{\sigma}$, $\delta\sigma = \sigma - \tilde{\sigma}$, and $\delta\omega_k = \delta\sigma_{2k}/H_{2k}^2$, $k = 1, 2, ..., n - 1$.

The direct and related problems are solved numerically using the finite-difference method with dimensions of the well disregarded. It is viewed as a point source with a power equal to the flow rate of the actual well [4]. Calculations for the model problems showed that the rate of convergence of the iteration process is linked with the selection of initial approximations of the hydraulic conductivities of weakly permeable seams and depends only slightly on initial approximations of the hydraulic conductivities for highly permeable seams. The practical selection of an initial approximation of the hydraulic conductivities for weakly permeable seams is accomplished as follows. At different values of the hydraulic conductivities of weakly permeable seams, 5–6 iterations are made, and thereafter, as approximate values of the hydraulic conductivities, such values are taken at which the residual as to bottom-hole pressures decreases most rapidly. The regularization parameter was chosen on the basis of the residual criterion [5]. Errors in measuring the discharges and bottom-hole pressures were 1–3%. When these errors were introduced in the initial data, the maximum error in determining the hydraulic conductivity in uniformity zones was 5%.

Further on, the proposed computational algorithm was used for interpreting the results of nonstationary hydrodynamic investigations obtained from well No. 1182 of the Romashkino field [6]. Well No. 1182 opens a bed with an



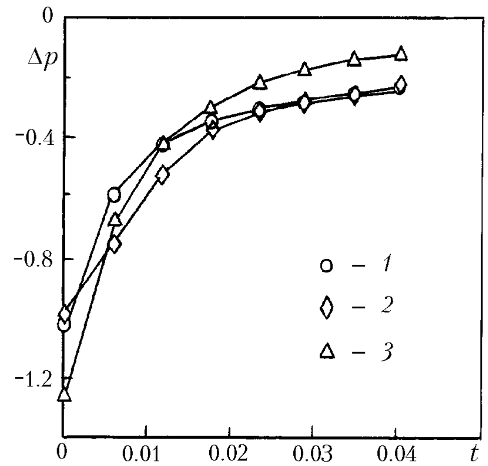


Fig. 1. Curves of variation in the persisting inflow in well No. 1182: 1) first seam; 2) second seam. q, m^3/day; t, day.

Fig. 2. Curves of variation in the bottom-hole pressures: 1) observed; 2) calculated for the first seam; 3) calculated for the second seam. Δp, MPa; t, day.

impermeable roof and a bottom divided by a weakly permeable dam. Graphs of the decrease in the discharges as a function of time (Fig. 1) and a curve of the variation in the bottom-hole pressure after the well is stopped were employed as the initial information. Filtration properties of the layered beds were evaluated assuming that, in the case of insignificant differences in the depths of occurrence of the seams, the time variation of the bottom-hole pressure in them is identical [1, 6]. Figure 2 presents calculated results, namely, the observed and calculated variations in the bottom-hole pressures. The hydraulic conductivities, obtained by graphic-analytical methods, for highly permeable seams are 0.245 and 0.365, respectively [6]. According to the proposed computational algorithm, these evaluations are 0.301 and 0.444.

NOTATION

A_i, component of a multidimensional quantity; $A\sigma$, implicitly specified nonlinear operator; c_1 and c_2, positive constants; D, filtration region; ds, element of length Γ; ∂D, boundary of the region; G, external boundary of the bed; H_{2k-1}, thickness of a highly permeable seam, m; H_{2k}, thickness of a weakly permeable seam, m; L_k, operator of the equation; $M^{\alpha}(\sigma)$, smoothing functional; $o(\cdot)$, symbol of the order; P^* and P^*_{δ}, multidimensional quantities; p_k, pressure in a highly permeable seam, MPa; p_{k0}, bed pressure in a highly permeable seam, MPa; $p_k^{(b)}(t)$, bottom-hole pressure, MPa; $\tilde{p}^{\,i}$, solution of the ith related problem; $q_k(t)$, discharge of a well, m^3/day; R^n, Euclidean space; T, time of the field experiment, day; t, running time, day; x and y, Cartesian coordinates, m; α, regularization parameter; β^*, elastic capacity coefficient, 1/MPa; Γ, circumference with a radius of 0.1 m; δ, measurement error; $\delta\sigma$, multidimensional quantity; $\delta\sigma_k$, increment in the hydraulic conductivity; $\delta\omega_k$, increment in the flow coefficient; φ_k, initial pressure distribution, MPa; σ, σ^m, and $\tilde{\sigma}$, multidimensional quantities; σ_{2k-1}, hydraulic conductivity of a highly permeable seam, $\mu m^2 \cdot m/(MPa \cdot sec)$; σ_{2k}, hydraulic conductivity of a weakly permeable seam, $\mu m^2 \cdot m/(MPa \cdot sec)$; σ_k^m, evaluations of the hydraulic conductivity in the mth iteration, $\mu m^2 \cdot m/(MPa \cdot sec)$; ω_k, flow coefficient; Δp, variation in the bottom-hole pressure; $\Omega(\sigma)$, Tikhonov stabilizer; $\partial/\partial n$, $\partial/\partial \tau$, and $\partial/\partial t$, derivatives with respect to the normal, the tangent, and time, respectively. Subscripts and superscripts: i, number of the related problem; k, seam number; m, iteration number; n, number of highly permeable seams; b, bottom-hole.

REFERENCES

1. S. G. Kamenetskii, V. M. Kuz'min, and V. P. Stepanov, *Oil-Field Studies of Beds* [in Russian], Nedra, Moscow (1974).

2. R. Earlougher, Jr., Advances in well test analysis, *SPE Monograph*, **5**, Dallas, SPE of AIME (1977).
3. M. Kh. Khairullin, Solution of inverse problems of filtration of multilayer beds by the regularization technique, *Dokl. Ross. Akad. Nauk*, **347**, 103–105 (1996).
4. K. Aziz and A. Settari, *Petroleum Reservoir Simulation* [Russian translation], Nedra, Moscow (1982).
5. O. M. Alifanov, E. A. Artyukhin, and S. V. Rumyantsev, *Extremum Methods for Solving Ill-Posed Problems* [in Russian], Nedra, Moscow (1988).
6. A. F. Blinov and R. N. Diyashev, *Study of Simultaneously Operated Beds* [in Russian], Nedra, Moscow (1971).